Classical Metatheory

Exercises 7: First-order predicate logic:
syntax, natural deduction, the axiomatic system QS

1. Given our definitions of ‘∨’, ‘∧’ for the language Q (lecture 4), and the following definition of the existential quantifier:
   \[ \exists v A =_{df} \neg \forall v \neg A \]
   (Compare Hunter, p. 139. Note that Hunter uses ‘W’ instead of ‘∃’.)
   Prove by induction over the complexity of formulae that Hunter’s language Q supplemented with these thus defined symbols and our language \( L^Q \) have exactly the same formulae (disregarding the abbreviations (a)–(f) introduced above).
   (Hint: It has to be shown that whenever \( A \) is a formula of Q, \( A \) is also a formula of \( L^Q \), and vice versa.)

2. Prove the equivalence of the natural deduction system for predicate logic introduced in lecture 7 and Hunter’s axiomatic system QS (§41), given our definitions of ‘∨’, ‘∧’ (lecture 4), and ‘∃’ (above), for the language Q.
   You may rely on the equivalence of the natural deduction system introduced in lecture 2 with Hunter’s system PS — this proof was given in lecture 4.
   (Hint: It suffices to add to both directions of the proof the clauses for the quantifiers.)